A non-destructive testing application solved with A-χ geometric eddy-current formulation

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Abstract

Purpose – The purpose of this paper is to introduce a perturbation method for the A-χ geometric formulation to solve eddy-current problems and apply it to the feasibility design of a non-destructive evaluation device suitable to detect long-longitudinal volumetric flaws in hot steel bars.

Design/methodology/approach – The effect of the flaw is accurately and efficiently computed by solving an eddy-current problem over an hexahedral grid which gives directly the perturbation due to the flaw with respect to the unperturbed configuration.

Findings – The perturbation method, reducing the cancelation error, produces accurate results also for small variations between the solutions obtained in the perturbed and unperturbed configurations. This is especially required when the tool is used as a forward solver for an inverse problem. The method yields also to a considerable speedup: the mesh used in the perturbed problem can in fact be reduced at a small fraction of the initial mesh, considering only a limited region surrounding the flaw in which the mesh can be refined. Moreover, the full three-dimensional unperturbed problem does not need to be solved, since the source term for computing the perturbation is evaluated by solving a two-dimensional flawless configuration having revolution symmetry.

Originality/value – A perturbation method for the A-χ geometric formulation to solve eddy-current problems has been introduced. The advantages of the perturbation method for non-destructive testing applications have been described.

Keywords Eddy currents, Testing, Steel

Paper type Research paper

I. Introduction

In non-destructive testing applications, there is a remarkable interest on the detection of defects, that can be located close to the surface, during hot mill rolling process of steel bars with circular cross-section (with a diameter from 8 to 80 mm, a speed from 5 to 100 m/s in the longitudinal direction z and a temperature from 800 to 1,200°C).

The capability to detect these flaws permits a fast and straightforward quality assessment of the product and provides the possibility to reduce those flaws due to

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a wrong setup of the manufacturing process parameters. The flaws considered have a depth ranging from 0.1 to 2 mm and, even though they have quite different shapes and sizes, they generally correspond to an interruption of the material continuity (also from the electrical point of view) and lay almost along an axial direction.

Two main categories of surface flaws can be considered, depending on their axial length L: the "short" flaws, with L ranging from 1 to 20 mm and the "long" flaws with L from a meter to tens of meters. Short flaws can be easily detected using a differential method in which the signal, after the noise reduction, is compared with a similar signal taken few centimeters away along the axial direction. On the contrary, so far, no practical solution has been found as regards the detection of long flaws, for which a differential approach is not suitable.

In this paper, the numerical modeling of the effect of long-insulating flaws is addressed when azimuthal eddy currents are induced in the bar. We will solve the resulting eddy-current problem by means of a "discrete geometric approach" (DGA) (Bossavit and Kettunen, 2000; Clemens and Weiland, 2001; Tonti, 1995), which allows to formulate Maxwell's equations in an alternative way with respect to the classical Galerkin method in finite elements.

II. The DGA for eddy currents

The domain of interest D of the eddy-current problem, has been partitioned into a source region D_s and in a passive conductive region D_c. The complement of $D_c \cup D_s$ in D represents the insulating air region D_a.

We introduce in D a pair of interlocked grids, the primal and the dual grid, one dual of the other. The incidence matrices relative to the primal and dual interlocked grids form the cell complexes $\mathcal{K}$ and $\mathcal{B}$, respectively. We denote the incidence matrices relative to $\mathcal{K}$ with $\mathbf{G}$ between edges e and nodes n, with $\mathbf{C}$ between faces f and edges e and with $\mathbf{D}$ between cells v and faces f (Bossavit and Kettunen, 2000; Clemens and Weiland, 2001; Tonti, 1995); the matrices $\tilde{\mathbf{G}} = \mathbf{D}^T$, $\tilde{\mathbf{C}} = \mathbf{C}^T$ and $\tilde{\boldsymbol{D}} = -\boldsymbol{G}^T$ describe the mutual interconnections of the dual complex $\mathcal{B}$.

The integrals of the electromagnetic field quantities with respect to the oriented geometric elements of the pair of complexes $\mathcal{K}$ and $\mathcal{B}$, are referred to as degrees of freedom (DoFs). Each DoF is stored in a DoFs array and indexed over the corresponding geometric element. The DoFs arrays will be denoted in boldface type.

We will use the following notation: the row of the array or matrix $\mathbf{x}$ relative to the geometric element k will be denoted by $(\mathbf{x})_k$. Moreover, the subscript s, c, a after an array or a matrix denote the sub-array or sub-matrix relative to entities belonging to D_s, D_c or D_a, respectively.

According to the Tonti's (1975, 1995, 1998) classification of variables, there is a unique association between every physical variable and the corresponding oriented geometric element.

A. A-χ geometric eddy-current formulation

In order to formulate an eddy-current problem using the A-χ formulation, the following DoFs arrays are introduced:

- $\mathbf{\Phi}$ is the array of magnetic fluxes associated with faces $f \in D$;
- $\mathbf{F}$ is the array of magneto-motive forces (m.m.f.s) associated with dual edges $e_{\mathcal{B}} \in D$;

- $\mathbf{I}$ is the array of currents associated with dual faces $f_{\mathcal{B}} \in D_c$. In D_s, we introduce the array $\mathbf{I}_s$ of impressed currents; and
- $\mathbf{U}$ is the array of electro-motive forces (e.m.f.s) on primal edges $e \in D_c$.

We introduce also the following arrays of DoFs relative to the potentials used in this formulation:

- $\mathbf{A}$ is the array of circulations of the magnetic vector potential $\mathbf{A}$ along the primal edges $e \in D$; and
- χ is the array of the electric scalar potential χ associated to primal nodes $n \in D_c$.

Maxwell's laws in the magneto-quasi-static limit can be written exactly as topological balance equations between DoFs arrays, as:

$$\begin{aligned} &(\mathbf{C}^T\mathbf{F})_e = 0, && e \in D_a \\ &(\mathbf{C}^T\mathbf{F})_e = (\mathbf{I}_s)_e, && e \in D_s \\ &(\mathbf{C}^T\mathbf{F})_e - (\mathbf{I})_e = 0, && e \in D_c, \end{aligned} \tag{1}$$

$$(\mathbf{\Phi})_f = (\mathbf{CA})_f, \quad f \in D, \tag{2}$$

$$\begin{aligned} &(\mathbf{G}_s^T\mathbf{I}_s)_n = 0, && n \in D_s \\ &(\mathbf{G}_c^T\mathbf{I}_c)_n = 0, && n \in D_c, \end{aligned} \tag{3}$$

where equation (1) is the Ampère's balance law, equation (2) involves the array $\mathbf{A}$ in such a way that Gauss' Law $\mathbf{D\Phi} = 0$ is satisfied identically (since $\mathbf{DC} = 0$). The continuity current balance law is discretized by equation (3).

Faraday's balance law in the frequency domain:

$$(\mathbf{C}_c\mathbf{U})_f = -i\omega(\mathbf{\Phi})_f, \quad f \in D_c \tag{4}$$

together with equation (2) yields:

$$(\mathbf{U})_e = -i\omega(\mathbf{A} + \mathbf{G}_c\chi)_e, \quad e \in D_c, \tag{5}$$

since $\mathbf{CG} = 0$ holds identically.

The discrete counterparts of the constitutive laws have to be considered:

$$\begin{aligned} &(\mathbf{F})_{e_{\mathcal{B}}} = (\nu\,\mathbf{\Phi})_{e_{\mathcal{B}}}, \quad e_{\mathcal{B}} \in D \ \text{(a)} \\ &(\mathbf{I})_{f_{\mathcal{B}}} = (\sigma\,\mathbf{U})_{f_{\mathcal{B}}}, \quad f_{\mathcal{B}} \in D_c \ \text{(b)}, \end{aligned} \tag{6}$$

where ν and σ are referred to as constitutive matrices, see for example Tonti (1995, 2002).

The final algebraic system, having $\mathbf{A}$ and χ as unknown DoFs arrays (Trevisan, 2004), can be written as:

$$\begin{aligned} &(\mathbf{C}^T\nu\,\mathbf{CA})_e = 0, && \forall e \in D_a \\ &(\mathbf{C}^T\nu\,\mathbf{CA})_e = (\mathbf{I}_s)_e, && \forall e \in D_s \\ &(\mathbf{C}^T\nu\,\mathbf{CA})_e + i\omega(\sigma\mathbf{A}_c)_e + i\omega(\sigma\,\mathbf{G}_c\chi)_e = 0, && \forall e \in D_c \\ &i\omega(\mathbf{G}_c^T\sigma\mathbf{A}_c)_n + i\omega(\mathbf{G}_c^T\sigma\,\mathbf{G}_c\chi)_n = 0, && \forall n \in D_c. \end{aligned} \tag{7}$$

As boundary conditions, we impose a zero $(\mathbf{A})_e$ on $e \in \partial D$ and we solve the singular linear system of equations without a gauge condition.

B. Constitutive matrices construction

The square matrix $\boldsymbol{\nu}$ (dim$(\boldsymbol{\nu}) = N_f$, N_f being the number of faces in D) is the reluctance matrix such that equation (6a) holds exactly at least for an element-wise uniform induction field B and magnetic field H in each cell and it is the approximate discrete counterpart of the constitutive relation $\mathrm{H} = \nu\,\mathrm{B}$ at continuous level, ν being the reluctivity assumed element-wise uniform.

The square matrix $\boldsymbol{\sigma}$ (dim$(\boldsymbol{\sigma}) = N_{e_c}$, N_{e_c} being the number of edges in D_c) is the conductance matrix such that equation (6b) holds exactly at least for an element-wise uniform electric field E and current density J in each cell and it is the approximate discrete counterpart of the constitutive relation $\mathrm{J} = \sigma \mathrm{E}$ at continuous level, $\boldsymbol{\sigma}$ being the conductance assumed element-wise uniform.

A classical way to construct the constitutive matrices $\boldsymbol{\nu}$ and $\boldsymbol{\sigma}$ for a tetrahedral mesh is the technique, described in Tarhasaari *et al.* (1999), where the resulting matrices are non-symmetric. The fact that matrices are non-symmetric is irrelevant for the reluctance matrix, while the conductance matrix $\boldsymbol{\sigma}$ can be constructed in a symmetric way (Specogna and Trevisan, 2005). Alternatively, the Galerkin Hodge technique (Bossavit, 2000) produces the same stiffness matrix as the finite elements with first order Whitney edge and face element basis functions. An original solution, that guarantees both stability and consistency[1] simultaneously, exploits a novel set of edge and face vector basis functions defined in Codecasa *et al.* (2007) for tetrahedra and triangular prisms.

For our application, a primal hexahedral grid will be particularly effective. In the case of hexahedral grids, the construction of the constitutive matrices can be addressed as in Dular *et al.* (2008), where a consistent but non-symmetric matrix is obtained. To gain symmetry, at the price of consistency, the Galerkin Hodge technique can be used instead, by means of the mixed elements vector base functions described in Dular *et al.* (1994). In order to recover again stability and consistency simultaneously, the original geometric technique presented in Codecasa *et al.* (2008) can be profitably used.

The main advantage of the DGA with respect to the correspondent finite element formulation is that the construction of the stiffness matrix is faster, since no numerical evaluation of a volume integral is needed, being the constitutive matrices constructed geometrically.

III. Application to non-destructive testing

The application concerns the design of a device for the detection of long-longitudinal flaws that can be present during the hot mill rolling process of the steel bars with circular cross-section.

The geometry of the unperturbed problem, shown in Figure 1, consists of a conducting AISI 310 steel bar, modeled as a conducting cylinder D_c. The radius of the bar is 17 mm and the conductivity is $\sigma = 1.236 \times 10^6\ S/m$.

A source coil D_s (32 mm inner radius, 40 mm outer radius, 80 mm height, and 640 turns) encircles the rod and is feeded by a sinusoidal current density of $10^6\,\mathrm{A/m^2}$ with a frequency of $f = 100$ kHz. Since the rod is hot, the lift-off is chosen to be 15 mm (Figure 2).

The perturbed problem is obtained from the unperturbed problem by considering in the bar a longitudinal perfectly insulating flaw D_f (Figure 3(a)), 0.5 mm deep from the surface of the cylinder and 0.2 mm thick (Figure 3(b)). The defect D_f is long, therefore it is present in all the longitudinal extension of the bar.

Unlike usual NDT methods that use global quantities, usually the impedance variation of the receiving coils, the sensors sample B on points evenly distributed over a circumference. We choose the circumference to lie in the plane $z = 40$ mm, with a radius of 31 mm and the center on the axis of the coil (Figure 2). The sampling circumference is considered fixed with the coil, since in practice the coil and the receiving sensors are assembled together.

The variation of the tangential component of B along the sampling points due to the presence of a flaw will be analyzed.

IV. A-χ for perturbed quantities

The variation of B due to the presence of a flaw may be obtained from the difference between the perturbed and unperturbed solutions. This approach presents various drawbacks.

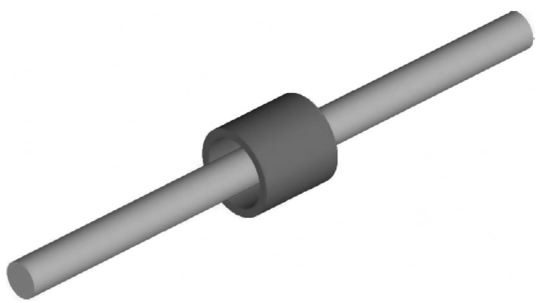


Figure 1.
The unperturbed eddy-current problem

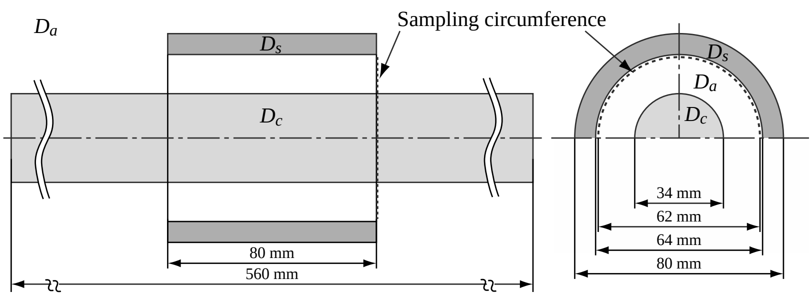


Figure 2.
The geometry of the unperturbed eddy-current problem

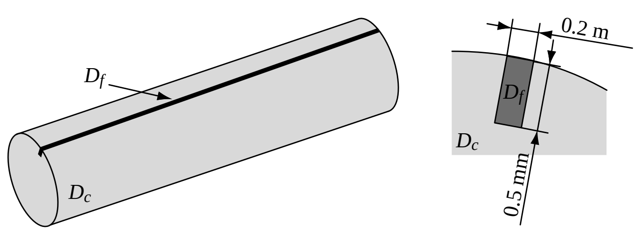


Figure 3.
The perturbed eddy-current problem is obtained from the unperturbed problem by considering a longitudinal flaw D_f

First of all, the full, three-dimensional, unperturbed problem has to be solved on exactly the same mesh of the perturbed problem, to minimize the error due to the mesh discretization. Moreover, the difference between the perturbed and unperturbed solutions may be affected by a considerable cancelation error.

Thanks to the linearity of the media involved, a problem which gives directly the perturbation due to the flaw can be formulated as (Albanese *et al.*, 1999; Sabariego and Dular, 2007):

$$\begin{aligned}
&(\mathbf{C}^T \nu\, \mathbf{CA})_e = 0, && e \in D_a \\
&(\mathbf{C}^T \nu\, \mathbf{CA})_e + i\omega\, (\sigma(\mathbf{A} + \mathbf{G}_c\chi))_e = 0, && e \in D_c \\
&(\mathbf{C}^T \nu\, \mathbf{CA})_e + i\omega\, (\sigma_f(\mathbf{A} + \mathbf{G}_f\chi))_e = -(\mathbf{I}_d)_e, && e \in D_f \\
&i\omega(\mathbf{G}_c^T \sigma\, (\mathbf{A} + \mathbf{G}_c\chi))_n = 0, && n \in D_c \\
&i\omega(\mathbf{G}_f^T \sigma_f(\mathbf{A} + \mathbf{G}_f\chi))_n = -(\mathbf{G}_f^T\mathbf{I}_d)_n, && n \in D_f,
\end{aligned} \tag{8}$$

where $\mathbf{I}_d$ is the array of source currents. The entries of the array $\mathbf{I}_d$ are computed from the unperturbed configuration; namely, they are the currents crossing the dual faces in the defect region D_f in the unperturbed problem.

Since the unperturbed configuration has revolution symmetry, its solution can be computed with high accuracy at reduced cost by solving a two-dimensional problem and by projecting it onto the dual faces of the dual mesh tailored within D_f. A reduced conductance in D_f has been assumed with respect to the conductance in D_c such that $\sigma_f/\sigma \approx 1,000$ holds.

A. Magnetic induction field computation

To calculate the value of the perturbation of induction magnetic field B components in a sample point P, two different approaches can be used:

(1) Find the hexahedron P belong to. Find the magnetic fluxes through its faces with $\mathbf{\Phi} = \mathbf{CA}$ and interpolate B(P) inside the hexahedron with the face-basis functions.

(2) Using the Biot-Savart Law:

$$\mathrm{B(P)} = \frac{\mu_0}{4\pi}\int_{D_c} \frac{\mathrm{J(P')} \times \hat{\mathrm{r}}}{r^2}\, dV, \tag{9}$$

where $\hat{\mathrm{r}}$ is the unit vector directed as the segment PP′.

The second solution is preferable, in fact it yields a more accurate value, being B the result of a global evaluation. The volume integral is computed using three Gauss' points.

V. Numerical results

As numerical example, a pair of structured primal meshes, with about 280 and 80k hexahedra yielding about one million of DoFs, respectively, have been used for the solution of the problem (8). The current sources in equation (8) are confined in the flaw region D_f, thus the meshes have been particularly refined in a neighborhood of the defect only (Figure 4(a)).

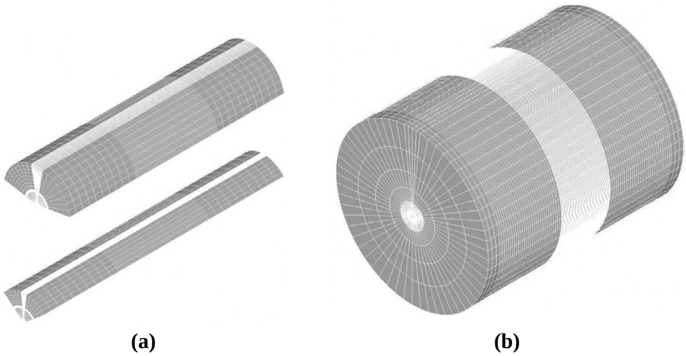


Figure 4.
The meshes used

Notes: (a) The formulation based on perturbation; (b) the solution based on abrupt difference between perturbed and unperturbed configurations

The source term has been computed by means of a 2D accurate axis-symmetric analysis, performed with a 2D version of the DGA on a 600k triangular elements primal grid.

The eddy currents computed in the perturbation problem (8) decrease rapidly away from the defect region. This can be seen from Figure 5, in which the real part of the eddy currents excited by feeding the defect is shown over across-section of the rod.

The azimuthal component B_t of the perturbation in the induction field around the rod due to the defect, is computed from the eddy currents by means of the Biot-Savart law (9) (Figure 6). The flaw is placed at 180° and produces a perturbation on the tangential B field that changes sign passing from one side of the flaw to the other.

As a comparison, the technique using the abrupt difference between the solutions of the defected and flawless configurations has been used. To this aim, the mesh in Figure 4(b) was used; it consists of 313k hexahedra (1.1 million of DoFs) and it is less refined nearby the defect region with respect to the meshes used with the perturbation method. This is because the elements have to be distributed uniformly in the skin depth of the conducting cylinder D_c, where the induced current density is confined.

In Figure 6, the variation ΔB_t in the real and imaginary parts of the tangential component of the magnetic induction due to the flaw obtained with the abrupt difference between perturbed and unperturbed solution are compared with the one

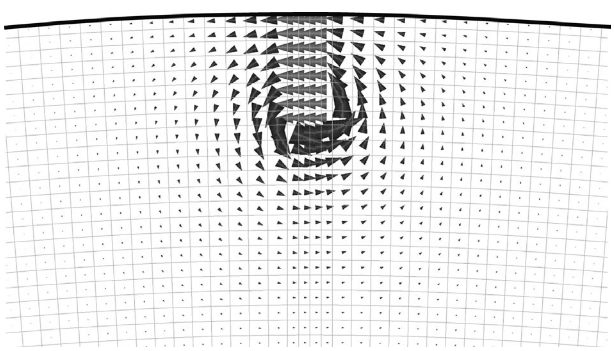
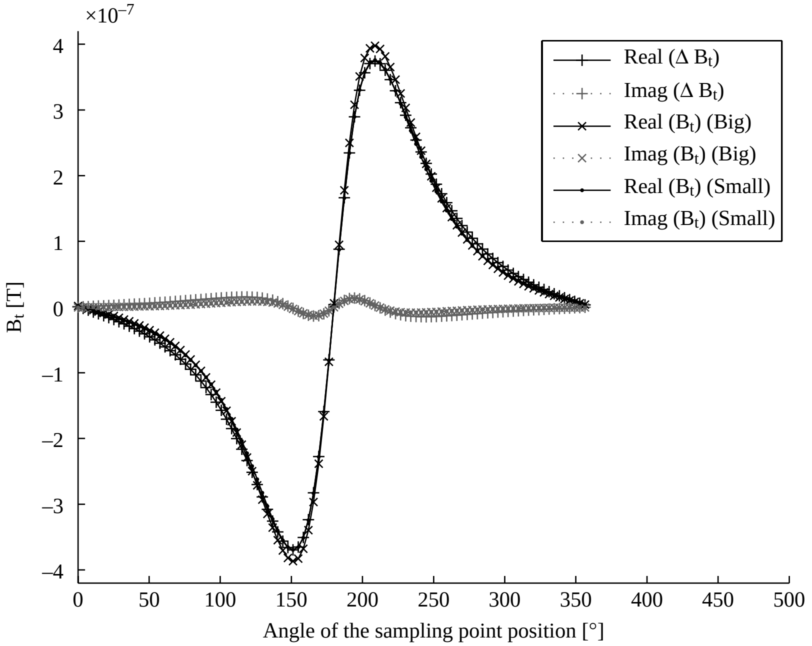


Figure 5.
The real part of the eddy currents resulting form the perturbation problem over the finer mesh is shown over a cross-section of the rod



Note: The results B_t obtained by perturbation method using the pair of meshes (denoted as BIG and SMALL) are compared with respect to the abrupt difference (ΔB_t)

Figure 6.
Tangential real and imaginary parts of the induction field on the sampling circumference around the bar

obtained by perturbation. The two alternative methods to compute such a variation are in good agreement. Nonetheless, the perturbation method allows to produce more accurate results at a less computational effort.

The accuracy improvement is due to the reduction of the cancelation error. This becomes fundamental especially when the formulation is used as a forward solver for an inverse problem. In this case, in fact, the variation at every iteration becomes smaller and smaller, requiring a good immunity to the cancelation error.

The perturbation method also reduces the computational time. First of all, with the abrupt difference, the full, three-dimensional, unperturbed problem has to be solved on the same mesh of the perturbed problem, to minimize the error due to the mesh discretization. Moreover, the linear system of equations resulting from the perturbation method converges more rapidly and no post-processing is needed for the unperturbed simulation. Moreover, the computation time of the source currents $\mathbf{I}_d$ for the perturbation problem is considerably smaller then the calculation of the source with an integral technique (Specogna and Trevisan, 2009) needed by the abrupt difference technique.

A further reduction of the computational time is obtained by observing that the source of the perturbation problem is localized into the flaw region. Consequently, the perturbation in the current density will be concentrated in a neighborhood of the flaw. For this reason, the initial mesh can be dramatically reduced considering only the hexahedra contained in a limited region surrounding the flaw (Figure 4(a)).

VI. Conclusions

The perturbation method, reducing the cancelation error, produces accurate results also for small variations between the solutions obtained in the perturbed and unperturbed configurations. This is especially required when the tool is used as a forward solver for an inverse problem. Moreover, the method yields also a considerable speedup: the mesh used in the perturbation problem can in fact be reduced at a small fraction of the initial mesh, considering only a limited region surrounding the flaw in which the mesh can be refined.

Note

1. A precise definition of the notion of consistency for constitutive matrices is given in Bossavit and Kettunen (2000).

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