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A boundary element method for eddy-current problems in fusion devices

Paolo Bettini [a,b,*], Maurizio Furno Palumbo [a], Ruben Specogna [c]

[a] Consorzio RFX, C.so Stati Uniti 4, 35127 Padova, Italy
[b] DII, Università di Padova, 35131 Padova, Italy
[c] DIEGM, Università di Udine, 33100 Udine, Italy

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ABSTRACT

In magnetic confinement fusion devices, close-fitting passive conducting structures surrounding the plasma are an efficient way to slow down the growth rate of magnetohydrodynamic (MHD) instabilities, enabling the possibility of an active stabilization by means of external actuators (coils). Nevertheless, a fully quantitative investigation, under realistic 3D boundary conditions, is still a very hard challenge. Moreover, the inclusion of fine geometrical details of actual passive structures might easily lead to impractical memory and computational time requirements if the problem is not carefully addressed. In this work we present an effective technique to solve eddy current problems in thin conductors of arbitrary topology by a boundary element method (BEM) based on a stream function. The proposed technique is applied to study the dynamic response of the stabilizing shell of RFX-mod, both to axisymmetric and non-axisymmetric external fields.

1. Introduction

Magnetically confined fusion plasmas are characterized, in many situations of interest, by the presence of unstable evolution modes [1,2]. The growth time of the ideal MHD instabilities is typically very short – of the order of microseconds for devices currently operating or under design – and would prevent any external corrective action. In practice, the eddy currents induced in the passive conducting structures surrounding the plasma slow down the growth rate of the instability to electromagnetic time scale – of the order of milliseconds or even slower for present devices and new machines under design – enabling the possibility of an active stabilization by means of external actuators (coils).

In modern experiments, excellent results have been achieved in the stabilization of both axisymmetric vertical instabilities, that can occur in elongated plasmas, and non-axisymmetric modes, that tend to deform the plasma boundary helicoidally.[1] In the latter case, external non-axisymmetric active coils (e.g. saddle coils) are required to produce a suitable magnetic field configuration to be coupled with the unstable modes.

However, a fully quantitative investigation, under realistic 3D boundary conditions, is still a very hard challenge, in particular when the closed loop analysis including feedback control is tackled. Moreover, the inclusion of fine geometrical details in the passive structure model (e.g. poloidal/toroidal gaps and apertures for diagnostics, vacuum and heating systems) may easily lead to unfeasible memory and computational time requirements if the problem is not carefully addressed.

In this work, we present an effective technique to solve eddy current problems in thin conductors of arbitrary manifold shape and topology by a boundary element method (BEM) based on a stream function.

Then, the proposed technique is applied to study the dynamic response of the stabilizing copper shell of RFX-mod, a medium size device ($R = 2$ m, $a = 0.459$ m) equipped with a complicated (geometrically and topologically) assembly and a state of the art feedback system. Both axisymmetric and non-axisymmetric external fields, in frequency domain, are considered.

2. BEM approach

When conductors have a thickness much smaller than the penetration depth of the magnetic field at the considered frequency, the induced current density can be considered as uniform across the layer thickness.

* Corresponding author. Tel.: +39 049 827 7545.

[1] The so-called kink instability has been one of the first and most crucial limiting instabilities encountered in magnetically confined fusion plasmas and it is still one of the most challenging issues in MHD stability and control for present machines [3].

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Efficient formulations to solve eddy current problems in thin conductors express the surface current density in terms of a scalar potential called *stream function*, see for example [4]. However, when the surface is not simply connected (because of the presence of handles and/or holes) the resulting stream function is multivalued and how to sort this issue out, in its generality, is a challenging task [5]. Moreover, heuristic approaches proposed in literature do not work with conductors as complicated (topologically, e.g. torus with many holes) as present in fusion applications.

In this work a boundary element method (BEM) based on a formulation suitable for multiply connected thin conductors is presented. The algorithm required in the topological preprocessing stage has been introduced in [6].

2.1. Formulation

Let us assume to have a mesh of the thin conductor formed by N nodes $\{n_i\}_{i=1}^N$, E edges $\{e_j\}_{j=1}^E$ and F faces $\{f_k\}_{k=1}^F$ whose incidences are encoded in the cell complex $\mathcal{K}$ [7]. With the standard barycentric subdivision, let us construct the dual nodes $\{\tilde{n}_k\}_{k=1}^F$, dual edges $\{\tilde{e}_j\}_{j=1}^E$ and dual faces $\{\tilde{f}_i\}_{i=1}^N$ that form the geometric elements of the dual complex $\tilde{\mathcal{K}}$, see Fig. 1a. Incidence matrix $\mathbf{G}$ stores the incidence between each edge and node pair. Moreover, one can easily verify that $\mathbf{G}^T$ gives the circulation of dual edges and dual node pairs.

Let us express the current per unit of thickness array of degrees of freedom (DoFs) $\mathbf{I}$ (we denote the jth DoF of $\mathbf{I}$ as I_j, see Fig. 1b), with

$$\mathbf{I} = \mathbf{G}\boldsymbol{\Psi} + \mathbf{H}\mathbf{i}, \qquad (1)$$

where $\boldsymbol{\Psi}$ is an array of DoFs whose coefficients are the values of the stream function on mesh nodes (see Fig. 1b), $\mathbf{i}$ is the array of *independent currents* [8] and the columns of $\mathbf{H}$ store the representatives of the so-called generators of a suitable cohomology group [6]. A fast graph-theoretic algorithm with a linear worst-case complexity to compute matrix $\mathbf{H}$ is introduced in [6] together with more insights on the physical interpretation of the cohomology generators. We remark that the algorithm is very easy to implement being based on computations of spanning trees, therefore the knowledge of cohomology theory is not necessary to implement or use it.

The coefficients of $\mathbf{I}$ relative to edges belonging to the boundary of $\mathcal{K}$ have to be set to zero since the current cannot cross the boundary of the conductor. In practice, this amounts to set the entries of $\boldsymbol{\Psi}$ corresponding to nodes belonging to $\partial\mathcal{K}$ to zero. We note that, for each connected component of $\mathcal{K}$ with empty boundary, one random coefficient of $\boldsymbol{\Psi}$ should be set to zero in order to obtain a symmetric and positive definite matrix.

Then, we enforce the discrete Faraday's law [7]

$$\mathbf{G}^T\tilde{\mathbf{U}} + i\omega\,\tilde{\boldsymbol{\Phi}} = -i\omega\mathbf{G}^T\tilde{\mathbf{A}}_s, \qquad (2)$$

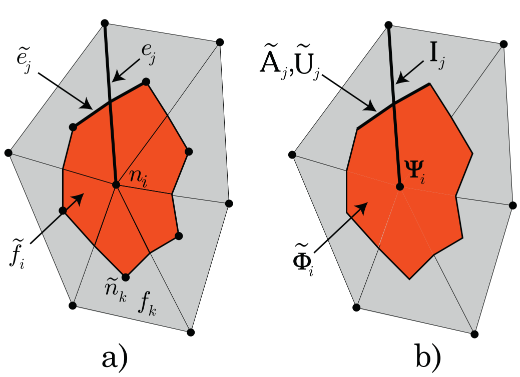

Fig. 1. (a) Geometric elements of the primal and dual complex. (b) Association of the physical variables to geometric elements of the primal and dual complex.

where $\tilde{\mathbf{U}}$ is the electromotive force on dual edges, $\tilde{\Phi}$ is the magnetic flux produced by eddy currents on dual faces and $\tilde{\mathbf{A}}_s$ is the circulation of the magnetic vector potential due to the source currents on dual edges. The discrete counterparts of the two constitutive laws are expressed by

$$\tilde{\mathbf{U}} = \mathbf{R}\mathbf{I} \quad \text{and} \quad \tilde{\mathbf{A}} = \mathbf{M}\mathbf{I}, \qquad (3)$$

where $\mathbf{R}$ and $\mathbf{M}$ are the resistance mass matrix and the magnetic matrix [4], respectively. By substituting (1), (3) and $\tilde{\boldsymbol{\Phi}} = \mathbf{G}^T\tilde{\mathbf{A}}$ inside (2) and by defining $\mathbf{K} = \mathbf{R} + i\omega\mathbf{M}$, one gets

$$(\mathbf{G}^T\mathbf{K}\mathbf{G})\boldsymbol{\Psi} + (\mathbf{G}^T\mathbf{K}\mathbf{H})\mathbf{i} = -i\omega\mathbf{G}^T\tilde{\mathbf{A}}_s. \qquad (4)$$

It turns out that the e.m.f.s on certain dual cycles (for example the ones that surround holes in the thin conductor) are not determined by a linear combination of the Faraday's laws (2). This is because there is no surface made by gluing dual faces whose boundary is such dual cycle. That means one needs additional constraints for such cycles. Bettini and Specogna [6] show that to obtain a square symmetric and positive definite system one has to write additional Faraday's laws enforced on suitable cycles that, thanks to the duality between $\mathcal{K}$ and $\tilde{\mathcal{K}}$, are obtained for free by using $\mathbf{H}^T$

$$\mathbf{H}^T\mathbf{K}\mathbf{G}\boldsymbol{\Psi} + \mathbf{H}^T\mathbf{K}\mathbf{H}\mathbf{i} = -i\omega\mathbf{H}^T\tilde{\mathbf{A}}_s. \qquad (5)$$

3. Numerical results

The proposed approach has been applied to study the dynamic response of thin conducting structures excited by external AC fields.

First, a hollow sphere in a sinusoidally varying magnetic field is considered. It is a convenient problem on which the proposed approach can be validated, since it has a well known analytical solution (see [4]). The eddy currents induced by a uniform vertical magnetic field (B_z = 1 T, f = 100 Hz) on a sphere made of a material with surface resistivity ρ = 1 μΩ and relative permeability μ_r = 1 are shown in Fig. 2. An excellent agreement is found with the analytical solution (see Fig. 3).

Then a torus with circular cross section, characterized by two gaps (one poloidal and one toroidal, on the high field side) is considered (see Fig. 4a). A more detailed geometry (see Fig. 4b) is also

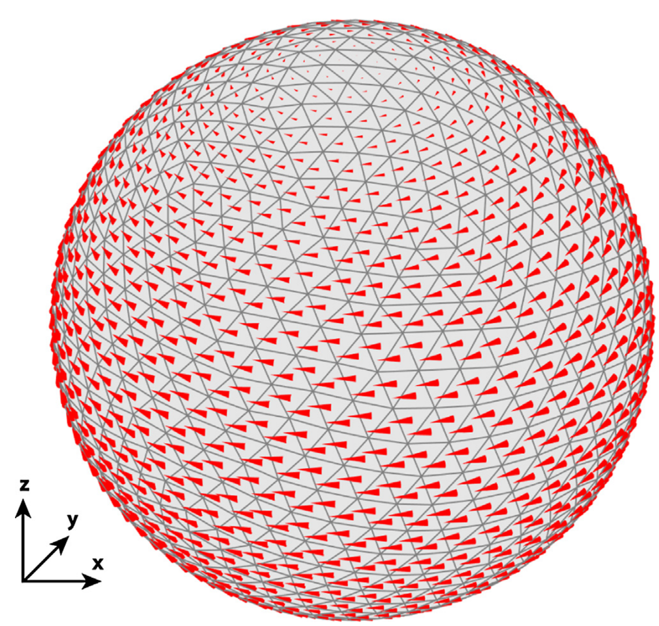

Fig. 2. Eddy currents induced by a vertical magnetic field on a hollow sphere discretized with 1888 triangles, 2832 edges, 946 nodes. Red cones: real part of the current density (a.u.), not to scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

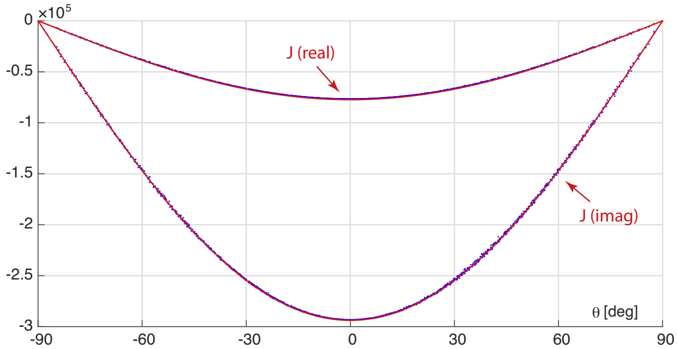

Fig. 3. Current density (real, imaginary) induced on the hollow sphere as a function of the azimuthal angle θ: comparison between the proposed approach (dots) and the reference solution (solid lines).

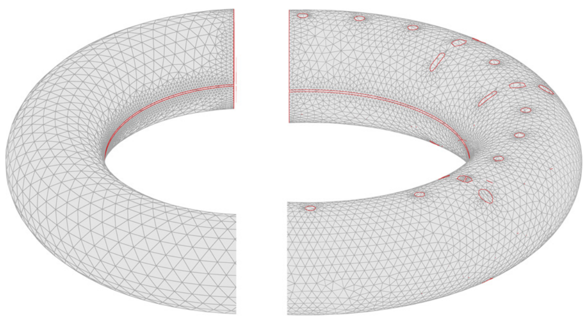

Fig. 4. (a) A torus with two gaps, discretized with 10,641 triangles, 16366 edges, 5726 nodes. Only half of the model is shown. (b) A torus with two gaps and several holes, discretized with 21,097 triangles, 32,776 edges, 11,606 nodes. Only half of the model is shown.

analyzed: the gaps and the port hole pattern resemble the one in RFX-mod, a medium size device (R = 2 m, a = 0.46 m) equipped with a complicated (geometrically and topologically) torus assembly (see Fig. 5) and a state of the art feedback control system [9,10].

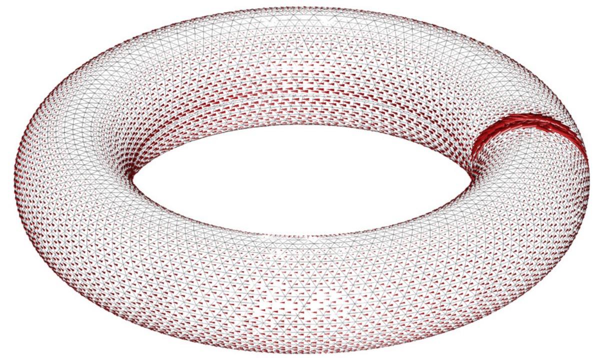

Fig. 6. Eddy currents induced in the first reference conducting structure by a uniform vertical magnetic field (sinusoid at f = 100 Hz). Red cones: real part of the current density (a.u.), not to scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The following test cases have been solved in the frequency domain:

1 Axisymmetric (vertical) magnetic field imposed by proper BCs (sinusoidally varying magnetic vector potential at f = 100 Hz).
2 Non-axisymmetric field produced by a local (saddle) coil (AC current at f = 50 Hz).

Fig. 6 shows the results of the analysis carried out on the first model (Fig. 4a) with axisymmetric excitation: the currents induced in the conducting structure exhibit the expected cosine pattern with respect to the poloidal angle and are constant along the toroidal direction, apart from the poloidal gap region where the 3D effect of the butt-joint configuration is clearly visible.

Then, the same axisymmetric excitation has been applied to the torus in Fig. 4b, to highlight the influence of fully 3D elements. A detail of this model is shown in Fig. 7: the deformation of the

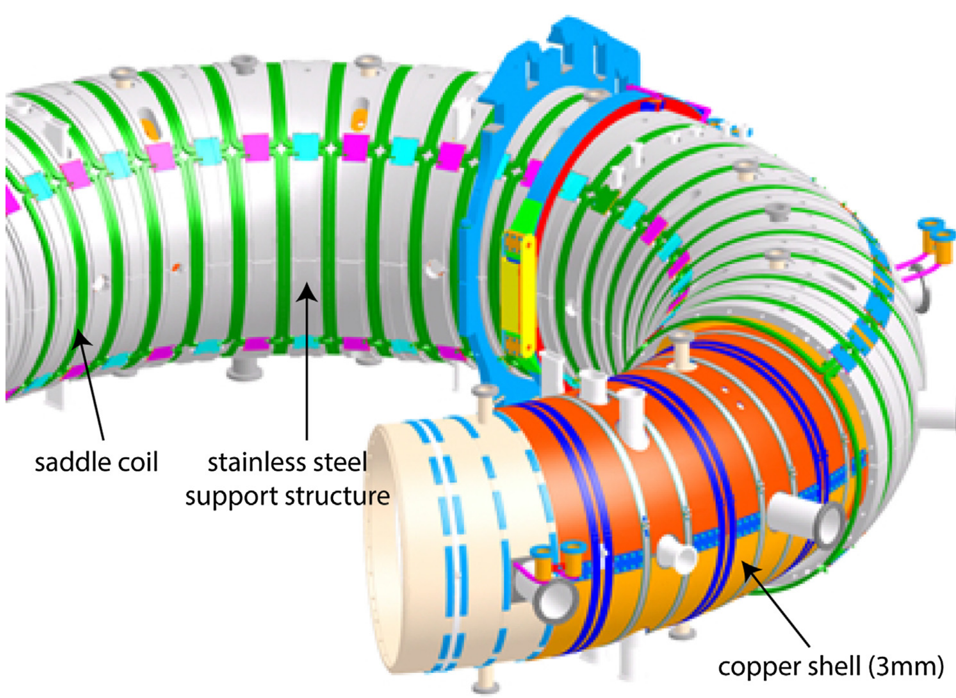

Fig. 5. Cutaway of the torus assembly of RFX-mod device. The main passive conducting structures (copper shell, stainless steel toroidal support structure) are shown together with a subset of the 192 (48 × 4) saddle coils, used for active control of MHD instabilities.

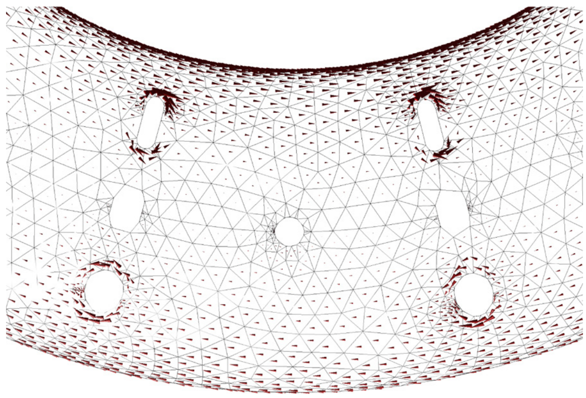

Fig. 7. Detail of the second reference model. The eddy currents are induced by a uniform vertical magnetic field (sinusoid at $f = 100$ Hz). Red cones: real part of the current density (a.u.), not to scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

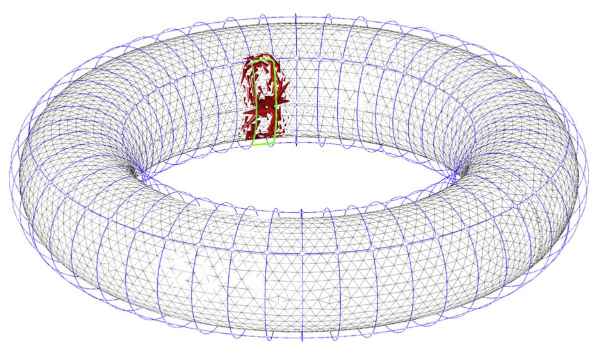

Fig. 8. Eddy currents induced in the first reference structure by a time varying magnetic field. The source is an inner coil (green) out of the 192 (48×4) saddle coils of the feedback system used to control MHD instabilities in RFX-mod. Red cones: real part of the current density (a.u.), not to scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

eddy current pattern around circular and oblong portholes is clearly visible.

Eventually, the proposed approach has been used to characterize the dynamic response of the conducting structures to a time varying excitation field, with a fully 3D structure, as the one typically produced by a saddle coil of the feedback system used to control MHD instabilities in RFX-mod [11]. The results shown in Figs. 7 and 8 prove that this approach allows to run electromagnetic simulations under the most realistic 3D excitations and boundary conditions.

Moreover, this approach is also reliable in terms of memory/computational time requirements. In fact, the solution of the linear system – Eqs. (4) and (5) – takes 0.3 s (1.2 s) for the first (second) model on a 2.6 GHz Intel Core i7, regardless of the kind of excitation; less than 7 GB (20 GB) of memory is required for the first (second) model solution.

4. Conclusions

An efficient boundary element method (BEM) based on a formulation suitable for multiply connected thin conductors has been presented and applied to study the dynamic response of a conducting structure (torus with circular cross section, two gaps and several portholes) which resembles the main stabilizing structure (copper shell) of RFX-mod device. The proposed approach proved to be very effective for the solution of fully 3D eddy current problems in frequency domain with minimum memory/computational time requirements.

Acknowledgment

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