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The AMDE project: 3D volumetric anomalies reconstruction by eddy current testing

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Abstract. The AMDE project ("Applicazioni di Metodi per Diagnostica Elettromagnetica", in English "Applied Methods for Electromagnetic Evaluation") is an Italian two-years national granted research project on electromagnetic non-destructive evaluation. This project involves five different research units located in Italy, about twenty researchers and ten among Ph.D. students and post-doc. Here we focus on the contribution of the AMDE project to the reconstruction of 3D volumetric defects in metallic plates as those used in aeronautical and aerospace industry. In particular, in this work we propose an efficient numerical model for evaluating the field due to volumetric anomalies and an inversion method based on the Total Variation Regularization. Numerical examples confirm the capability of the proposed strategy to reconstruct 3D volumetric defects.

Keywords: Eddy current testing, numerical methods, total variation regularization, inverse problems

1. Introduction

Non-destructive Imaging of Materials is a field of researches that is continuously gaining attention for its intrinsic potential. Here we focus our attention on electromagnetic imaging of conductive materials by means of Eddy Current Testing (ECT) inspection [7].

In ECT systems a low-frequency time-varying magnetic field induces eddy currents in a conductive material. Anomalies in the material change locally the electrical conductivity and affect the magnetic reaction field. ECT of conductive materials can be applied to a great variety of different problems such as, for instance, periodic maintenance operations for airplane or nuclear power plants, oil/gas pipelines inspection, etc.

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Actually, commercial systems have the capability of detecting the presence of defects (cracks, voids, etc.) in a conductor and may, eventually, provide a estimate of the most relevant sizes. Despite of this, state-of-the-art commercial ECT systems are not capable of forming an image of the anomalies. This is due to the extremely complex relationship between the measured data and the electrical conductivity, thus attracting the interest of several research groups from different countries. This scenario is the reference frame for the AMDE project, mainly focused on the development of methods, algorithms and techniques toward the realization of electromagnetic non-destructive systems with quantitative (imaging) capabilities suitable for real world problems.

The "ingredients" needed to achieve this goal are three: (i) fast and accurate numerical models to predict the response of an ECT probe, (ii) fast and accurate numerical methods to process the measured data to estimate position and shape of the defect(s), (iii) optimized ECT probes with high sensitivity to the typical "target" anomaly.

The paper first summarize some results achieved by the AMDE project on the eddy current testing of conductive materials (Section 2), then it presents a specific contribution on the reconstruction of three-dimensional defect in a conductive plate (Section 3) and finally it draws some conclusions.

2. Algorithms and techniques developed during the AMDE project

Fast and accurate numerical models [6,9,10,12] to predict the response of an ECT probe are essential for both the computer aided design (CAD) of the probe and the processing of the measured data. Regarding the first issue, it is worth noting that the sensitivity of commercial available probes to defect sizes is generally low but for "large" defects. Therefore, the probes have to be optimized [3]. To this purpose, computer aided design is a valuable tool to increase the effectiveness of the probe, as well as, to reduce the cost and the development time. Accurate and fast numerical models of the probe-specimen interaction are also essential to develop inversion algorithms capable of forming an image of the interior of the conductor from the measured data [4,13]. State-of-the-art quantitative imaging algorithms for solving this ill-posed non-linear problem are based on the comparison between the measured data and the numerically computed data associated to trial anomaly configurations. These algorithms usually require a large number of comparisons and, since each computation requires the solution of a magneto-quasi-stationary problem, this task can be extremely time-consuming preventing its application in a real world scenario.

Similarly, it is very important to develop ad-hoc imaging algorithms that exploit all specific features of the problem (ECT) under investigation. General purpose algorithms are unsuitable in this context because of their large computational time (mainly due to the need for solving repeatedly the forward problem). Ad-hoc iterative methods [10] or non-iterative imaging algorithms [14], where the image of the interior of the body is obtained by doing the aforementioned comparison with "only" a number of pre-computed numerical data that is of the order of the pixels (voxels) representing the image, represent an effective solution to this issue.

3. Reconstruction of 3D volumetric anomalies in metallic plates

In this section we focus on the solution of the inverse problem of retrieving volumetric defect in metallic plates by means of eddy current testing (see Fig. 1). This approach combines an efficient numerical model developed during the AMDE project with a total variation regularization algorithm [10].

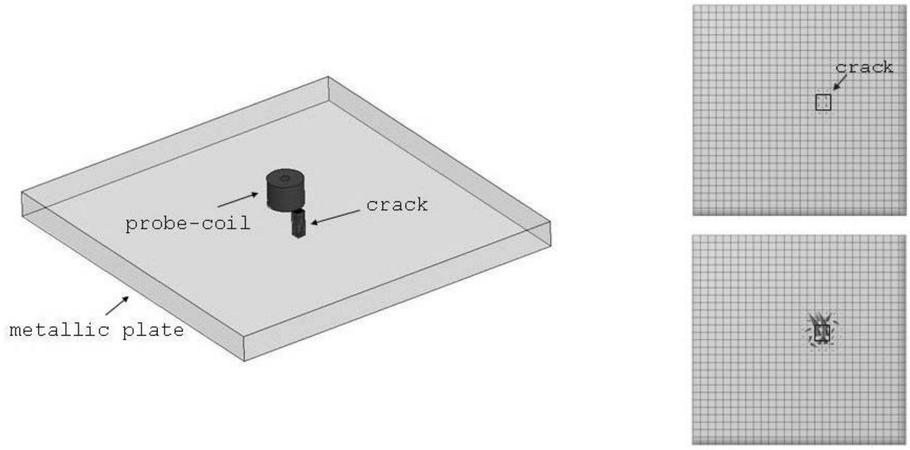

Fig. 1. Left: a typical eddy current testing setting. Right: an example of perturbed eddy current density $\Delta \mathbf{J}$ flowing locally (top: real part, bottom: imaginary part).

The inversion algorithm is extremely versatile thanks to its capability of reconstructing an arbitrary number of defects having arbitrary topology. The penalty term introduced by the total variation regularization, is particularly suited for reconstructing piecewise constant functions. This method is not indicated when a defect is not homogeneous (from the macroscopic viewpoint).

3.1. The forward model

The numerical method here presented has been developed to fully exploit all specific features intrinsic in the modeling of the interaction of an eddy current testing coil with a material that presents one or more defects. The exploiting of all specific features of the problem is, in the authors' viewpoint, not an option but a mandatory step to develop a reliable and efficient numerical code to be used together with a quantitative imaging algorithm.

The most important specific feature in ECT is that the typical defect occupies a small volume of the conductor under testing and, therefore, the defect perturbs the induced eddy current density locally. In the context of our numerical model, that is an integral formulation in term of the electric vector potential, this translates in assuming as unknown the perturbation $\Delta \mathbf{J}$ of the eddy current density due to the presence of the defect. Another important feature is that, by means of simple data processing algorithm, it is rather easy to find a candidate region, the so-called tentative region V_T, where the defect is supposed to be present. The tentative region is, obviously, larger than the defect but, in almost all practical configurations, is significantly smaller than the test specimen.

Let η and η_0 be the electrical resistivity of the conductor with and without the defect(s), respectively. In the absence of magnetic materials, in the low frequency limit, the eddy current density perturbation $\Delta \mathbf{J}$ satisfies the following problem:

$$\eta \Delta \mathbf{J} + j\omega \mathbf{A} \left[\Delta \mathbf{J} \right] = -\Delta \eta \mathbf{J}_0 - grad \left(\Delta \varphi \right), \text{ in } V_C \quad (1)$$

where $\Delta \mathbf{J} \in \left\{ \mathbf{w} \in L^2_{div} \left(V_C^{LOC} \right) | \nabla \cdot \mathbf{w} = 0 \text{ in } V_C^{LOC},\ \mathbf{w} \cdot \hat{\mathbf{n}} = 0, \text{ on } \partial V_C^{LOC} \right\}$, V_C^{LOC} is a neighborhood of the defect where $\Delta \mathbf{J}$ is non vanishing (see Fig. 1), $\Delta \eta = \eta - \eta_0$ is the resistivity perturbation representing the defect, $\mathbf{J}_0$ is the unperturbed eddy current density, $\Delta\varphi$ is the scalar potential perturbation, V_C is the conductive domain and the operator $\mathbf{A}$ is defined as:

$$\mathbf{A}:\mathbf{v}(\mathbf{r})\rightarrow\frac{\mu_0}{4\pi}\int\limits_{V_C}\frac{\mathbf{v}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V. \tag{2}$$

The following numerical model, presented in [10], is based on the framework proposed in [1] for solving eddy currents problems. Specifically, the unknown is represented as $\Delta\mathbf{J}=\sum_{k\in AE}\Delta I_k\nabla\times\mathbf{N}_k$ where $\mathbf{N}_k$ is an edge-element shape function, and *AE* is a subset of the edges of the graph made by nodes and edges of the finite element mesh that, when properly chosen, allows to impose automatically the gauge and the boundary condition.

The numerical model is easily obtained by imposing Eq. (1) in weak form:

$$\int_{V_C^{LOC}}\nabla\times\mathbf{N}_k\cdot(\eta\Delta\mathbf{J}+j\omega\,\mathbf{A}[\Delta\mathbf{J}])dV=-\int_{V_C^{LOC}}\nabla\times\mathbf{N}_k\cdot\Delta\eta\mathbf{J}_0dV\quad\forall\mathbf{N}_k, \tag{3}$$

where we have exploited that the term involving $\Delta\varphi$ gives no contribution thanks to the solenoidality of the test functions and their vanishing normal component on the boundary. The corresponding linear system is:

$$(\mathbf{Z}_0+\Delta\mathbf{R})\,\Delta\mathbf{I}=-\Delta\mathbf{R}\mathbf{I}_0 \tag{4}$$

where, $\mathbf{Z}_0=\mathbf{R}_0+j\omega\mathbf{L}$, $\Delta\mathbf{I}$ is the column vector of the coefficients representing the expansion of $\Delta\mathbf{J}$ and

$$\mathrm{L}_{\mathrm{ij}}=\frac{\mu_0}{4\pi}\int\limits_{V_C^{LOC}}\int\limits_{V_C^{LOC}}\frac{\nabla\times\mathbf{N}_\mathrm{i}(\mathbf{x})\cdot\nabla\times\mathbf{N}_\mathrm{j}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}dV\,dV',\quad I_{0,k}=\int\limits_{V_C^{LOC}}\nabla\times\mathbf{N}_k\cdot\mathbf{J}_0\mathrm{d}V \tag{5}$$

$$\mathrm{R}_{0,\mathrm{ij}}=\int\limits_{V_C^{LOC}}\nabla\times\mathbf{N}_i\cdot\eta_0\nabla\times\mathbf{N}_j\,dV,\quad\Delta\mathrm{R}_{\mathrm{ij}}=\int\limits_{V_C^{LOC}}\nabla\times\mathbf{N}_i\cdot\Delta\eta\nabla\times\mathbf{N}_j\,dV \tag{6}$$

Once the induced current density perturbation $\Delta\mathbf{J}$ has been computed, it is possible to compute the related voltage induced on the coil. It is worth noting that: (i) the use of a local discretization V_C^{LOC} of the conductive material allows to achieve a great reduction in the computational cost [9,10] and (ii) the unperturbed current density $\mathbf{J}_0$ can be computed analytically for canonical geometries.

In Eqs (3), (5) and (6) V_C^{LOC} is a proper neighborhood of the tentative region V_T $\left(V_T\subset V_C^{LOC}\subseteq V_C\right)$. In this way, for a given defect contained in the tentative region, only matrix $\Delta\mathbf{R}$ needs to be recomputed. Discretizing a neighborhood of the tentative region instead of the anomaly, may be not so efficient when the anomaly occupies only a small part of V_T. However, this is essential when, as in an iterative imaging algorithm, the responses associated to many different trial defect configurations, that may occupy completely different regions of V_T, have to be evaluated.

3.2. *The inversion algorithm*

Let the measurement be the value of time-harmonic (complex) voltages induced on the exciting coil due to the presence of the anomalies, collected at different spatial locations and frequencies.

The inversion algorithm here discussed is based on the minimization of the discrepancy regularized with the total variation regularization [10] term:

$$TV\left(\mathbf{x}\right)=\int_{V_T}\left|\nabla\sigma\right|dV \tag{7}$$

where $\mathbf{x}$ is the column vector containing the parameters describing the electrical conductivity $\sigma=\eta^{-1}$, here assumed piecewise uniform (x_i is the value of the electrical conductivity in the i-th "block"). The total variation regularization term penalizes functions with great spatial variability but preserving the edges [11,15,16] and, therefore, it is appropriate when the unknown is "blocky", as in the present problem. The inverse problem is, therefore, cast as the minimization of:

$$E\left(\mathbf{x}\right)=\left|\mathbf{V}^{*}-\mathbf{V}\left(\mathbf{x}\right)\right|^{2}+\alpha TV\left(\mathbf{x}\right) \tag{8}$$

where $\mathbf{V}^{*}$ and $\mathbf{V}\left(\mathbf{x}\right)$ are the measured and numerically computed voltages variations due to the presence of anomalies and α is the regularization parameter.

The implemented inversion algorithm update iteratively the current solution $\mathbf{x}^{(n)}$ at step n with the following rule:

$$\mathbf{x}^{(n+1)}=\mathbf{x}^{(n)}+P\Delta\mathbf{x}^{(n)} \tag{9}$$

where P is a projection operator and $\Delta\mathbf{x}^{(n)}$ minimizes

$$E^{(n+1)}\left(\Delta\mathbf{x}\right)=\left|\mathbf{V}^{*}-\mathbf{V}\left(\mathbf{x}^{(n)}\right)-\mathbf{S}^{(n)}\Delta\mathbf{x}\right|^{2}+\alpha TV\left(\mathbf{x}^{(n)}+\Delta\mathbf{x}\right), \tag{10}$$

$\mathbf{S}^{(n)}$ being the Jacobian arising from the linearization of $\mathbf{V}\left(\mathbf{x}\right)$ in a neighborhood of $\mathbf{x}^{(n)}$.

The projection operator P takes into account the a priori information that the electrical conductivity cannot be negative or larger than the value σ_b of the host material. Specifically, the i-th component of $P(\mathbf{x})$ is:

$$\left(P\left(\mathbf{x}\right)\right)_i=\begin{cases}0, \text{ for } x_i<0\\ x_i, \text{ for } 0\leq x_i\leq\sigma_b\\ \sigma_b, \text{ for } x_i>\sigma_b\end{cases}. \tag{11}$$

The minimization of $E^{(n+1)}$ is carried out iteratively by means of the *Lagged Diffusivity Fixed Point Iteration Method* introduced by Vogel and Oman [15,16].

As well known, the choice of a proper value for the regularization parameter α is a critical issue. We found numerically that the L-curve method is satisfactory for this class of inverse problems.

Measurements from different frequencies may have different magnitude. Therefore, the data, consisting in multi-frequency measurements, have to be properly weighted. To accomplish this aim, we operated a frequency dependent normalization of $\mathbf{V}$ and $\mathbf{S}$ based on a SVD analysis of the sensitivity matrix at single frequencies and on the knowledge of the data noise variance.

3.3. Numerical results

The test case presented in this section refers to an Aluminum plate (thickness 3 mm, $\sigma=37.7\times10^{6}$ S/m).

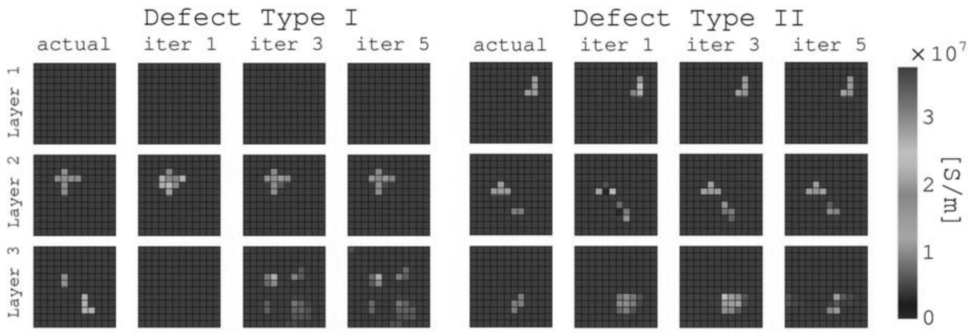

Fig. 2. Results of the inversion. The columns '*iter n*' represent the n-th iteration. The noise level is $\varepsilon = 5 \cdot 10^{-6}$ corresponding to 4.07% for Type I defect and 2% for Type II defect.

The tentative region V_T where the electrical conductivity has to be reconstructed is a 12 mm $\times$ 12 mm $\times$ 3 mm region. The tentative region is subdivided in 12 $\times$ 12 $\times$ 3 cubic cells having edges of 1 mm. In each cell the unknown electrical conductivity is assumed to be uniform.

The probe consists of a coil with the following characteristics: inner radius 0.5 mm, outer radius 2 mm, height 3 mm, 200 turns, lift-off 0.5 mm. The probe is placed in 25 different positions, whose centres are located onto a regular grid of points oriented at 45 degrees respect to the x axis and equally spaced at a distance of about 2 mm that corresponds to the coil outer radius. In this way, the probe covers the tentative region in a satisfactory manner. A multi-frequency excitation is exploited in order to inspect the inner structure of the plate at different depths. We adopt three different values (670, 1500, 6000 Hz) whose skin-depths correspond to the layer depths (1 mm, 2 mm and 3 mm).

To avoid the so called "inverse crime" and to analyze a realistic configuration, we have corrupted the numerically computed measurements with a random generated noise. Specifically, to each single (synthetic) measurement is added a random having a maxim amplitude given by $\varepsilon V_{\mathrm{air}}$ where ε is the noise level and V_{air} is the voltage across the coil when placed in air. In the numerical example $\varepsilon = 5 \cdot 10^{-6}$. This value is in line with measurement instruments having an accuracy of 5 digits. It is worth noting that the noise level in term of percentage of the measured quantity (the coil voltage variation due to the defect only) is much higher because the coil voltage variation due to the defect is only a fraction of V_{air}. In particular, using the norm for the discrepancy (first term at the r.h.s. of Eq. (8)) the noise on the measured quantity is 4.07% (2%) the Type I (Type II) defect presented in Fig. 2.

Figure 2 shows the spatial distribution of some reconstructed electrical conductivity. As expected, the defect in the inner-most layer are more difficult to be retrieved. Nevertheless the results show that the electrical conductivity spatial distributions are satisfactory assessed and, moreover, the number of required iteration is low (5 iterations).

4. Conclusion

In this paper we have given a brief overview of the contributions made during the AMDE project, a two years long project carried out by five different research units located in Italy. Moreover, we have presented the contribution given by this project on the specific problem of locating three-dimensional defects in conductive plates by eddy current testing. The proposed method process measurements of the voltage variation across a coil and induced by the presence of the defect. The measurements are collected at different spatial locations and at different frequencies. The inversion method is an iterative method where both the underlying inversion algorithm and forward model are optimized for the specific setting. The overall inversion method results to be accurate and fast in term of required iterations for getting the solution.

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