Goal-Oriented Adaptivity for Voltage Breakdown Prediction

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An innovative approach has been recently proposed for the voltage breakdown prediction in high-voltage systems, insulated by large vacuum gaps. This approach is based on complementary geometric formulations for electrostatics coupled to the analytical solution of the equations of motion for charged particles. In this paper, a goal-oriented local mesh refinement technique is introduced, which allows to increase the rate of convergence of the solution, enabling an effective voltage breakdown prediction also in large-scale systems with complex geometries.

Index Terms— **Charged particles, complementary formulations, electrostatics, goal-oriented mesh adaptivity.**

I. Introduction

AN ORIGINAL technique has been recently proposed by Pilan *et al.* [1], Bettini *et al.* [2], and Marconato *et al.* [3] for the voltage breakdown prediction in high-vacuum devices extensively used in many research and industrial areas. This method aims to improve the design of such complex electrostatic devices with respect to the simple analysis of the electric field distribution, on the basis of a more advanced physical model, namely the Slivkov–Cranberg clump theory [4]. Developed for the design of the electrostatic accelerator of the neutral beam injector for the International Thermonuclear Experimental Reactor [1], [2] having the challenging requirement of withstanding 1 MV in a very tight high vacuum environment, it was later adopted for the optimization of Siemens medium-voltage vacuum circuit breakers (VCBs) [3]. The proposed method has the twofold advantage of allowing on the one hand identifying the most critical regions of the considered geometry, which can be more prone to breakdowns, on the basis of the above-mentioned criterion, and on the other hand to identify a univocal probabilistic relation with respect to the voltage applied.

In many industrial problems, one is interested in approximating some function of the solution, known as the quantity of interest, rather than the solution itself. This is exactly the case of the problem addressed in this paper, where the quantity of interest is the breakdown probability as a function of the voltage P.

In this case, the so-called *goal-oriented adaptivity* [9] yields to a more efficient convergence with mesh refinement than standard adaptivity based on complementarity [5]–[7], given that the enrichment of degrees of freedom is focused on the improvement of P rather than the electrostatic energy. Given a set of particle trajectories from the cathode, P is a function of the triple product W, which is a local variable that in turn depends on the electric field at the origin and at destination of the trajectory [see eq. (3)]. The rigorous theory of the goal-oriented adaptivity presented, for example, in [9] cannot be exploited in our case, because the relation between the quantity of interest and the solution is non-linear, in general discontinuous, and it is available only implicitly and inexactly through the computation of particle trajectories and the subsequent evaluation of the triple product W.

This is why, in this paper, we propose a more heuristic solution inspired from the constitutive inconsistency that [10] introduces an error indicator for full Maxwell problems for which, similarly to our case, rigorous error estimators are hard to devise. The idea here is to refine the elements with the biggest difference in the triple product (W_V, W_{MH}) computed with the formulation based on the electric scalar potential (**V**) and the mixed-hybrid one (**M**–**H**), respectively, that should provide the same solution instead. This method does not provide guaranteed error bounds for P, yet it has a clear physical ground and this inconsistency between the two formulations can be used as an error indicator. Using the costly mixed-hybrid formulation based on the electric displacement vector field (**d**) is also justified, because it provides accurate trajectories thanks to the enforcement of the continuity of the normal component of the field through faces of the mesh elements [8].

After a brief description of the breakdown prediction model and a short survey about complementary formulations for electrostatics, this paper introduces the novel idea of a goal-oriented adaptivity cycle based on a physical-grounded error indicator devised using the same spirit of the constitutive error of [5]. This paper ends with some numerical results to show the sensible speed-up obtained by the proposed method with respect to the standard complementarity-based adaptivity [5]–[7] and finally draws some conclusions.

II. Voltage Breakdown Prediction Model

The voltage breakdown prediction model is a statistical approach based on the following assumption. It considers a system with a number N of electrodes polarized at different voltages V_i and a number of micro-particles n_i per surface unit, which can be detached by electrostatic forces from the

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generic electrode acting as a cathode and clash to the receiving electrode with sufficient energy to vaporize.

The breakdown probability associated to the area A_i of the ith elementary subdivision (mesh element) of the cathodic electrodes is expressed by

$$p_i = n_i A_i. \tag{1}$$

Using the same approach followed by the failure analysis theory [11], the breakdown probability for the whole system is simply

$$P = 1 - \prod_{i=1}^{M} (1 - p_i) \tag{2}$$

being M the overall number of electrode subdivisions.

The crucial point of the model is to identify a relationship between n_i and the physical mechanism underlying the clump-induced breakdown. According to the Slivkov–Cranberg criterion [4], a breakdown can occur if the triple product W associated with the generic micro-particle

$$W = \Delta V E(P_1) E(P_2)^{2/3} \tag{3}$$

exceeds a threshold W_s, being $E(P_1)$ the norm of the electric field at the starting point P_1 (responsible for clump charging at electrode 1 - cathode), $E(P_2)$ the norm of the electric field at the destination point P_2 (responsible for the vapor bubble ionization at electrode 2 - anode), and ΔV the voltage between the two electrodes.

A fundamental assumption is that the number of micro-particles that potentially can produce a breakdown is a monotonic function of W, and, in particular, a functional dependence described by Weibull's distribution [12] is assumed

$$n(W) = n \left(\frac{W}{W_0} \right)^m. \tag{4}$$

The final expression of the breakdown probability, associated with the studied device, is a Weibull's distribution function depending on m and W_0 parameters, which are the characteristics of the electrode material and surface state, thus experimentally obtained, and on the cumulative contribution of all the possible trajectories, weighted by their corresponding triple product W.

A. Calculation of Particle Trajectories

Given a certain mesh of an electrode, all trajectories starting from the barycenters of its faces are calculated (see an example in the schematic geometry represented in Fig. 1). This step constitutes the core of the application and its correctness strongly influences the final result in terms of breakdown probability. The knowledge of the electric field distribution in the whole domain is in turn required, whose precise calculation has, therefore, to be assured.

An efficient technique for the calculation of the particle trajectory has been applied, which does not make use of the Runge–Kutta integration scheme, since it solves the whole particle motion by the analytical solution inside each element and moves from an element to the adjacent one. By adopting the first-order elements, the electric field E inside each cell is uniform and, thus, a charged particle moving inside the generic tetrahedron is subjected to a uniformly accelerated motion, which can be easily calculated analytically [2].

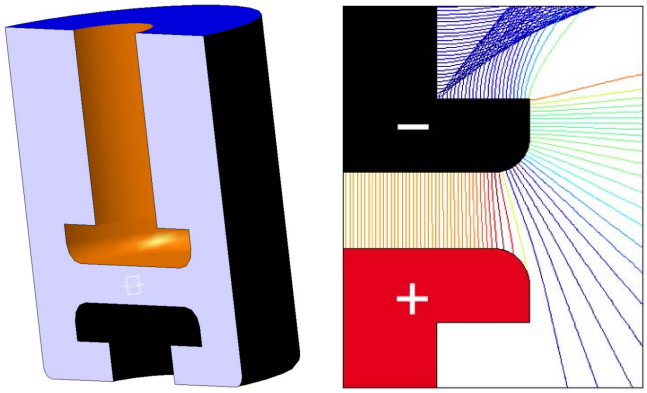

Fig. 1. Left: schematic of a VCB. Right: examples of possible trajectories (from cathode to anode) calculated in a 2-D axisymmetric configuration. Each trajectory is tagged with a color from a color map corresponding to W values. Blue line: min. Red line: max.

B. Formulations

Here, we explain our choices related to the formulations used by the proposed technique. First, we consider only the first-order elements for good reasons. As a matter of fact, the geometry of the electrodes always exhibits a strong curvature that has to be efficiently and correctly represented in the mesh if one wants to really achieve the predicted asymptotic order of convergence of a high-order method. The same issues arise when corner or edge singularities are present, which is always the case in industrial applications, and they have to be correctly captured given that the voltage breakdown probability results depend mostly on them. Thus, even using a high-order method would require to use millions of elements anyway, defeating even the convergence of the extremely high-order state-of-the-art schemes like [14]. Finally, the first-order schemes are amenable to the technique in [2] for particle tracking, which is much faster and more accurate than the Runge–Kutta scheme.

For the aforementioned reasons, we decided to use two first-order complementary formulations [5], [6] to guide the automatic mesh refinement: the standard scalar potential formulation (**V**) and the mixed-hybrid (**M**–**H**). We use the **M**-**H** formulation [6] as the formulation that is conservative in the electric flux, because it is usually the fastest among the other possibilities [13]. We remark these formulations to produce trajectories of much better quality [8] than the ones based on the solution based on the electric scalar potential formulations.

C. Goal-Oriented Automatic Mesh Adaptivity

The automatic mesh refinement based on the classical complementarity is based on the so-called constitutive error [5], [6] and speeds up sensibly the convergence for the electrostatic energy and all other global variables related to it. Unfortunately, when applied to our problem, it usually does not produce a convergence that is fast enough for practical applications in terms of holding probability.

Following the same philosophy behind the constitutive error [5], the novel error indicator is built from the mismatch in the quantity of interest produced for a given mesh by the

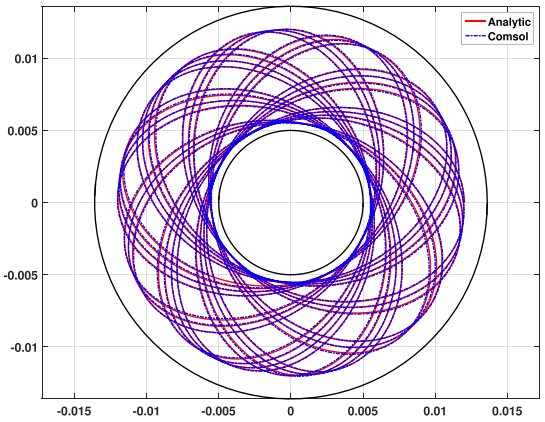

Fig. 2. Orbital trajectory of a charged particle in a 2-D electrostatic coaxial configuration ($r_1 = 1$ cm, $r_2 = 2.7$ cm, and $\Delta V = 1$ kV). Trajectory calculated with the analytic method (CAFE, red line) and numerical integration scheme with tolerance $\epsilon = 10^{-6}$ (Comsol, blue line).

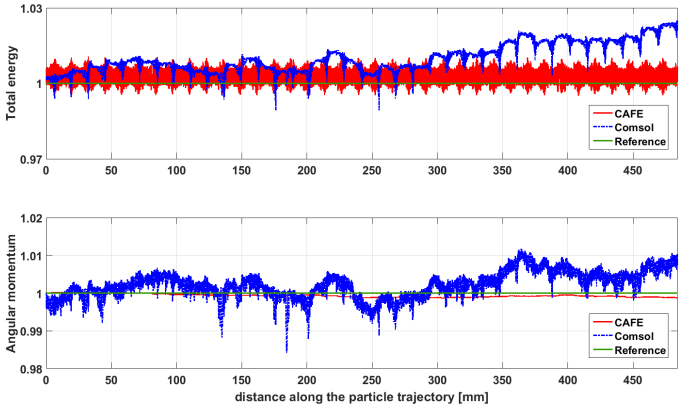

Fig. 3. Total energy W_t and angular momentum L, as a function of the curvilinear abscissa along the trajectory, calculated with the analytical approach (red line) and the numerical integration scheme (blue line) implemented in Comsol ($\epsilon = 10^{-6}$).

two complementary formulations. In our case, the quantity of interest are clearly all triple products W as defined in (3) evaluated on all electrode faces. The philosophy is, in fact, that the two formulations give exactly the same results when the mesh grain is pushed toward zero, thus refining those elements with the biggest mismatch in the quantity of interest cannot fail to improve the accuracy of simulation [6]. In practice, we found that this improvement speeds up sensibly the convergence of the holding probability.

We found good results by refining both the volumetric elements with the highest (top 5%) constitutive error λ and the surface elements, covering the electrodes, with the highest (top 5%) error λ', being

$$\lambda = \frac{|\mathbf{d}_{\mathrm{MH}} - \varepsilon \mathbf{e}_V|^2}{2\varepsilon}, \quad \lambda' = \frac{|W_{\mathrm{MH}} - W_V|^2}{2} \tag{5}$$

where $\mathbf{d}_{\mathrm{MH}}$ is the electric flux density computed with the M–H formulation and $\mathbf{e}_V$ is the electric field computed with the V formulation.

III. Numerical Results

Two benchmark electrostatic problems have been carried out first for assessing the robustness and reliability of the proposed particle tracing algorithms.

The first benchmark consists of the calculation of the orbital trajectory of a charged particle in the 2-D electrostatic coaxial configuration shown in Fig. 2, carried out with the proposed semi-analytical method (red line) and by standard numerical integration in Comsol (blue line).

In this planar central force field, the total energy W_t (sum of the kinetic and electric potential energy) and the angular momentum L are conserved along the charged particle trajectory. Fig. 3 shows the (normalized) values of W_t and L along the trajectory calculated with the two methods. Table I summarizes the results in terms of global errors (max and rms) for different values of the integration tolerance (ε) set in Comsol.

By normalizing the values of W_t and L to the values corresponding to the initial conditions ($t = 0$), every variation of the calculated quantities along the particle trajectories corresponds to an error of the solution. The results shown in Fig. 3 prove

TABLE I
Errors on Normalized Energy and Angular Momentum Values

	W - error		L - error	
	rms	max	rms	max
CAFE algorithm (analytic)	0.00379	0.01082	0.00191	0.00352
Comsol ($\epsilon = 10^{-4}$, default)	0.07969	0.22930	0.07002	0.18316
Comsol ($\epsilon = 10^{-5}$)	0.02340	0.10716	0.02404	0.09237
Comsol ($\epsilon = 10^{-6}$)	0.01099	0.02516	0.00382	0.01587

the superior quality of the trajectories calculated with the semi-analytical approach implemented in the numerical code CAFE.

The second benchmark consists of the calculation of the trajectories of charged particles moving in the radial field between two spherical shells (inner and outer electrodes). For this configuration, an analytic solution does exist (the trajectory consists of an ellipsis, or part of it), which allows to perform a pointwise comparison between the calculated trajectories and their (analytical) reference. The problem is solved in CAFE with a 3-D model (the numerical domain is covered by $\approx 10^6$ tetrahedra) without imposing any symmetry condition. In Fig. 4, two representative trajectories are shown (the charged particle can reach the external shells or fall again in the internal one on the basis of the relative value of the initial radial velocity with respect to the voltage applied): in both the cases, the maximum discrepancy between the trajectory computed in CAFE and the reference one is of the order 1 mm (i.e., $<1\%$ with respect the problem radial dimension).

Then, the proposed procedure is applied to the calculation of the voltage breakdown probability for a reference VCB geometry. A 3-D solution is obtained in CAFE (Fig. 5) and validated against the 2-D axisymmetric solution obtained with the electric vector formulation (**P**) introduced in [7]. In Fig. 6, the results obtained by the new mesh adaptivity scheme (solid lines) and those calculated by the previous one (dashed lines) are compared. The plot on the top shows that regarding the quantity related to the electrostatic energy, as the electric charge Q, the two adaptivity methods provide equivalent results, since there is no significant difference in terms of convergence efficiency. On the contrary, regarding a specific objective function as the voltage breakdown probability is,

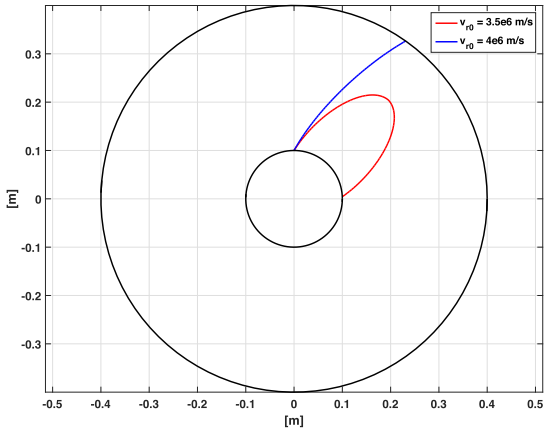

Fig. 4. Trajectories of charged particles in the radial field between two spherical shells ($r_1 = 10$ cm, $r_2 = 40$ cm, and $\Delta V = 100$ kV) calculated with CAFE: starting at r_1 with tangential velocity $v_t = 2.2\ 10^6$ m/s and two values of the radial component: 3.5 10^6 m/s (red line) and 4 10^6 m/s (blue line).

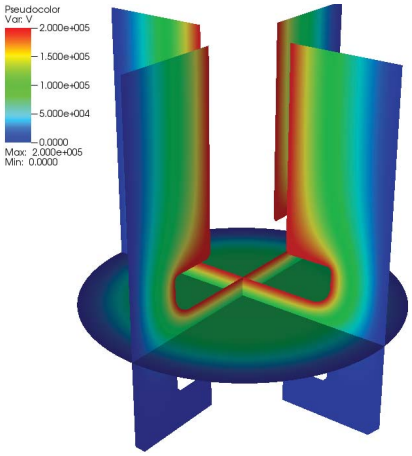

Fig. 5. Electric potential distribution for a reference VCB configuration by the CAFE code: electrode gap $g = 10$ mm and applied voltage $V = 200$ kV.

the proposed goal-oriented adaptivity based on the use of a suitable combination of the standard and the specific error indicator shows better performances in terms of convergence efficiency. Furthermore, in Fig. 6, the results obtained by the **V** and **M–H** formulations are shown in green lines and blue lines, respectively.

We remark that in the **V** formulation the electric field is discontinuous between elements sharing a face even if they have the same material parameter. It is, therefore, impossible to construct a streamline crossing these two elements with a classical method based on particle tracking [8]. That is why the discharge probability produced by the scalar potential formulation should always be considered less accurate than the one produced by the **M–H** formulation, which also motivates the interest in the latter formulation.

IV. Conclusion

This paper shows that the combined use of the mixed-hybrid formulation together with the proposed semi-analytical approach allows an efficient calculation of high-quality charged particle trajectories. In addition, it has been shown that the use of complementary formulations allows capturing

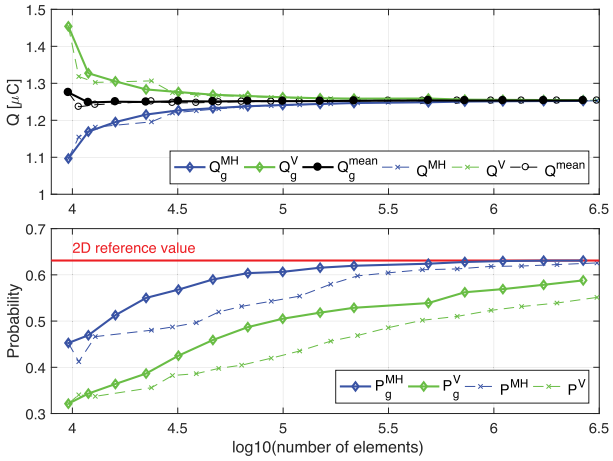

Fig. 6. Rate of convergence of the electric charge deposited on the positive electrode (top) and voltage breakdown probability (bottom). The results of scalar potential formulation (**V**) and mixed-hybrid formulation (**MH**) calculated by the new mesh adaptivity scheme (solid line, subscript g) are compared to those calculated by the previous one (dashed line).

field singularities thanks to the automatic mesh refinement. Moreover, by adopting the concept we called *goal-oriented adaptivity*, a convergence speed up is evident.

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